Indian Statistical Institute, Bangalore
M. Math First Year
Second Semester - Differential Geometry I

Back Paper Exam — Date: June 14, 2018
Maximum Marks: — Duration: 3 hours

Attempt all questions, they carry equal marks.

1. Let $\alpha(t)$ be a unit speed curve in $\mathbb{R}^3$ with curvature $k(t) \neq 0$ for all t. Define a new curve δ by $\delta(t) = \frac{d\alpha(t)}{dt}$. Show that $\delta(t)$ is regular. If s is the arc length parameter for s, show that $\frac{ds}{dt} = k$ and the curvature of s is $\sqrt{1 + \frac{t^2}{k^2}}$ where t is the torsion of α.

2. Let σ be a surface patch with N as its standard unit normal. Let $F_3 = \begin{pmatrix} \|N_u\|^2 & N_u.N_v \\ N_u.N_v & \|N_v\|^2 \end{pmatrix}$. Let F_1, F_2 denote the matrices of the first and second fundamental forms of σ respectively. Show that $F_3 = F_2 F_1^{-1} F_2$.

3. Let H and K denote the mean and Gaussian curvatures of a surface patch σ. Let F_1, F_2, F_3 be as in (2). Show that $F_3 - 2HF_2 + KF_1 = 0$.

4. Let $F : [0, \pi] \to \mathbb{R}$ be a smooth function with $F(0) = F(\pi) = 0$. Prove that $\int_0^\pi F'^2 dt \geq \int_0^\pi F^2 dt$ and equality holds if and only if there exists a real number A such that $F(t) = A\sin(t)$ for all $t \in [0, \pi]$.